(Marks: 2)
Find the degree sequence of the following graph

(Marks: 2)
Let A and B be events with

$P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$ and $P(A\cap B)=\frac{1}{4}$

Find

$P(A|B)$

Solution:

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

$$=\frac{\frac{1}{4}}{\frac{1}{3}}$$

$$=\frac{1}{4}\times\frac{3}{1}=\frac{3}{4}$$

(Marks: 3)

Find the greatest common divisor of the following pair of integer:
72,63

Solution:

1.Divide 72 by 63:
This gives 72 = 63 * 1 + 9
2.Divide 63 by 9:
This gives 63 = 9 * 7 +0

Hence gcd (72, 63) = 9.

(Marks: 2)
Find all non isomorphic simple connected graphs with three vertices.

(Marks: 3)
How many 3-digit numbers can be formed by using each one of the digits 2,3,5,7,9 only once?
Solution:
5*4*3=60

(Marks: 3)
How many permutations of the letter of the word PANAMA can be made, if P is to be the first letter in each arrangement?

Solution:

Total letter =6

Like letters = A =3

First letter is P already selected , remaining =5

Therefore,

P (5,3) = 6

(Marks: 5) Incomplete Question

A die is weighted so that the outcomes produce the following probability distribution:

Outcome	1	2	3	4	5	6
Probability	0.1	0.3	0.2	0.1	0.1	0.2

Consider the event

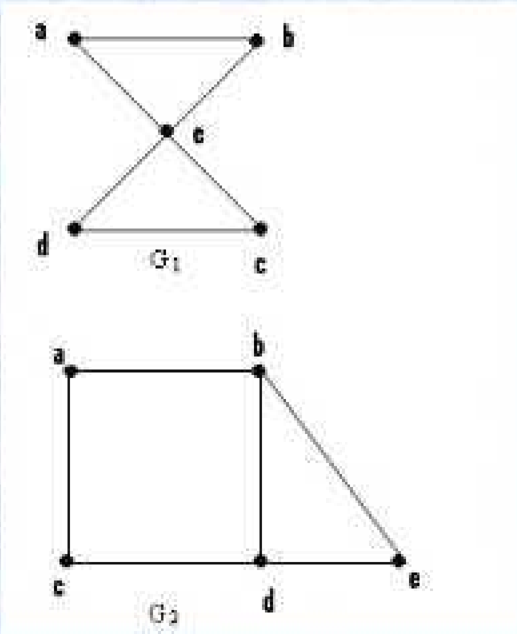
A= {even number} then find the following

a) P(A)

b) $P(A^c)$

(Marks: 5)

Determine whether the given graphs have an Euler circuit? If it does, find such a circuit, if it does not, give an argument to show why no such circuit exists.



(Marks: 5)

By using Mathematical induction prove that for all positive integers n

$1^2 + 2^2 + 3^2 + ... + n^2 = n(n+1)(2n+1)/6$

(Marks: 10)

Prove by mathematical induction that $3^{(2n-1)} + 1$ is divisible by 4 for all $n \geq 1$.

Question No: 21 (Marks: 2)

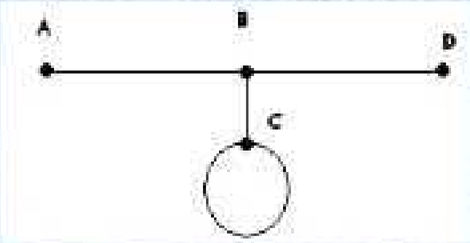
Find integers q and r so that a=bq+r, with $0 \leq r < b$.

a=45 , b=6.

Solution:

If a =45 and b = 6 are two integers with b ≠ 0 such that the q and r are non negative integers.
a=bq+r
divides 45 by 6
this gives= 6*7 +3
divides 6 by 3
this gives = 3*2 +0
hence gcd of the (45,6) will be 3

Question No: 22 (Marks: 2)
Give the degree of each vertex in the figure (given below)



Solution:

degree of A vertex = 1
Degree of B vertex = 3
Degree of C vertex = 3
Degree of D vertex = 1
Total degree of vertices = 8
Can be prove by formula
Degree of vertices = 2. no. of edges
= 2 . 4
= 8

Question No: 23 (Marks: 2)
What is the probability of getting a number greater than 2 when a dice is tossed?

Solution:
As dice has 6 sides so possible event will be 36.
No. greater than 2 will be
3 ,4 ,5 ,6 = 4 outcomes are greater than 2

P(E) =n(E)/(n(S))

= 4/36

= 1/9 will be the possibility to get no greater than 2

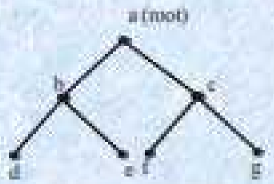
Question No: 24 (Marks: 3)
How many distinguishable ways can the letters of the word HULLABALOO be arranged if words are to begin with U and end with L

Solution:
If the words are to begin with U and end with L, then there are eight positions left to fill.
Where,
There are 3 L alike
2 O alike
2 A alike.
Therefore, the permutation becomes.

$$=\frac{8!}{3!*2!*2!}=\frac{40320}{24}=1680$$

Question No: 26 (Marks: 3)
Draw a full binary tree with seven vertices.
exact example hai Lecture 44 ki
Solution:
EXERCISE:
Draw a full binary tree with seven vertices.
SOLUTION:
Total vertices= 2k + 1 = 7 (by using the above theorem)
$\Rightarrow$ k = 3

Hence, total number of internal vertices (i.e. a vertex of degree greater than 1)=k=3
and total number of terminal vertices(i.e. a vertex of degree 1 in a tree) =k+1=3+1=4
Hence, a full binary tree with seven vertices is



Question No: 27 (Marks: 5)

Find n if
P(n,2) = 72

Solution:

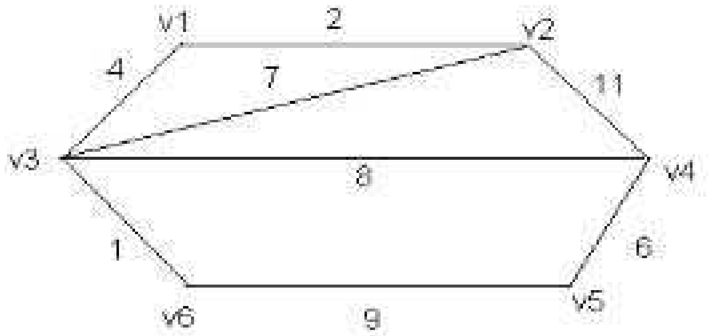
Given
P(n,2) = 72
n.(n-1)=72 by using the definition of permutation
n2-1=72
n2-n-72=0
n=9,-8 since n must be positive so only the acceptable value for n is 9

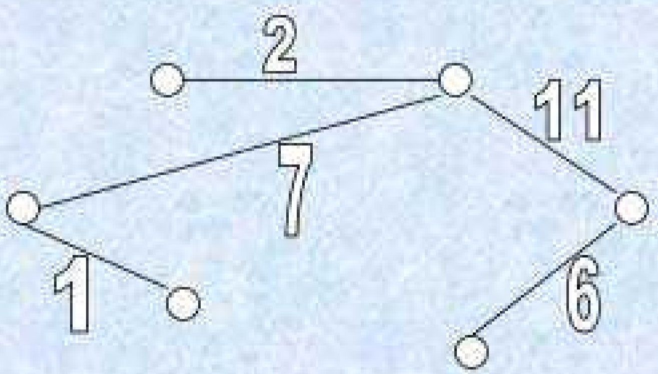
Question No: 28 (Marks: 5)

Five people are to be seated around a circular table. Two seating plans are considered as same if one is the rotation of other. How many different seating plans are possible?

Question No: 29 (Marks: 5)

Use Kruskal's Algorithm to draw the minimal spanning tree for the graph below. Indicate the order in which edges are added to form a tree.





{v3,v6},{v1,v2},{v4,v5},{v2,v3},{v2,v4},.

Question No: 30 (Marks: 10)
Show the sample space for tossing one penny and rolling one die.
(H = heads, T = tails) using tree diagram

Question No: 31 (Marks: 10)
$10^{3n} + 13^{n+1}$ is divisible by 7 for all n>=1

Solution:

Let $10^{3n} + 13^{n+1}$ is divisible by 7

Basis step:
P(1) is true.now
P(1):

(1.1)

$10_{3n}+13_{n+1}$ is divisible by 7

Since $10^{3\cdot1}+13^{1+1} = 10^3+13^2$
This is divisible by 7
Hence P(1) is true.now

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Inductive step:

Suppose p(k is true)

$10_{3k} + 13\ k_{+1} = 7.q$

To prove p(k+1) $10_{3n} + 13_{n+1}$ is true is divisible by 7

$10_{3k+1} + 13_{k+1+1} = 23_{4k+3}$

$= 23_{4k+3} + 2 - 2$

$= 21_{4k+3} + 2$

$= 7.3_{4k+3} + 2$

$= 7(3_{4k+3} + 2)$

$= 7.\ q$ where q is ant positive integer equal to $3_{4k+3} + 2$

So its proved that $10^{3n} + 13^{n+1}$ divisible by 7 for all n>=1

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